MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2018/2019

EME3156 - COMPUTATIONAL METHODS FOR MECHANICAL ENGINEERING

(ME)

30 MAY 2019

9.00 a.m.– 11.00 a.m.

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 8 pages including the cover page.
2. Answer all questions.
3. The distribution of the marks for each question is given.
4. Please write all your answers in the answer booklet provided.
5. All necessary workings **MUST** be shown.

Question 1

Equation (1.1) shows a first order ODE with initial condition,

$$\frac{dy}{dx} = y - 2sinx \quad , y(0) = 1 \tag{1.1}$$

When $x = 0.25$ and 0.50, the exact values for y are 1.216 and 1.357 respectively.

a) By using midpoint method with step size $h = 0.25$, find $y(0.5)$. [6 marks]

b) By using fourth order Runge-Kutta (RK4) with step size $h = 0.5$, find $y(0.5)$. [7 marks]

c) Comment on both methods used to solve the problem. [2 marks]

Question 2

A nonlinear system of equations is given:

$$\begin{aligned} x_1^3 - 5x_2^2 + 7 &= 0 \\ x_1 x_2^2 - 1 &= 0 \end{aligned} \tag{2.1}$$

Let $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and $\overline{F}(x) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix}$ where $\overline{F}$ is a vector-value function of a vector argument $\boldsymbol{x}$. The original nonlinear system can be expressed as the vector equation $F(x) = 0$. 0 is a zero vector.

a) Find the Jacobian matrix for the system, $\left[J\left(x^{(0)}\right)\right]$ when the initial guess is given as $x^{(0)} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$. Determine also $F\left(x^{(0)}\right)$. [7 marks]

b) Determine the improved approximation $x^{(1)}$. [8 marks]

Continued...

Question 3

Equation (3.1) shows a second order ODE with the boundary conditions.

$$\frac{d^2y}{dx^2} - y = x, \quad y(0) = -1 \qquad y(0.8) = 0.527 \tag{3.1}$$

a) By using central difference, convert equation (3.1) into explicit difference form. [2 marks]

b) Step size of $h = 0.2$ is suggested. The domain of the problem is discretised into 5 nodes. Incorporating boundary conditions, rearrange the difference equations from (b) into matrix form, $[K]\{y\} = \{F\}$, where

$[K]$ is a square matrix
$\{y\}$ is a vector containing all unknown nodes (y_2, y_3, y_4)
$\{F\}$ is a vector containing unknown nodal displacement [10 marks]

c) Use Gauss Elimination method to solve the matrix equation. [8 marks]

Continued...

Question 4

Temperature distribution of a two-dimensional irregular metal can be expressed in terms of a steady state diffusion equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{f(x,y)}{a}=0 \qquad (4.1)$$

where $f(x,y)$ is the power density at (x,y) and a is the thermal conductivity. For this problem, $\frac{f(x,y)}{a}=0$

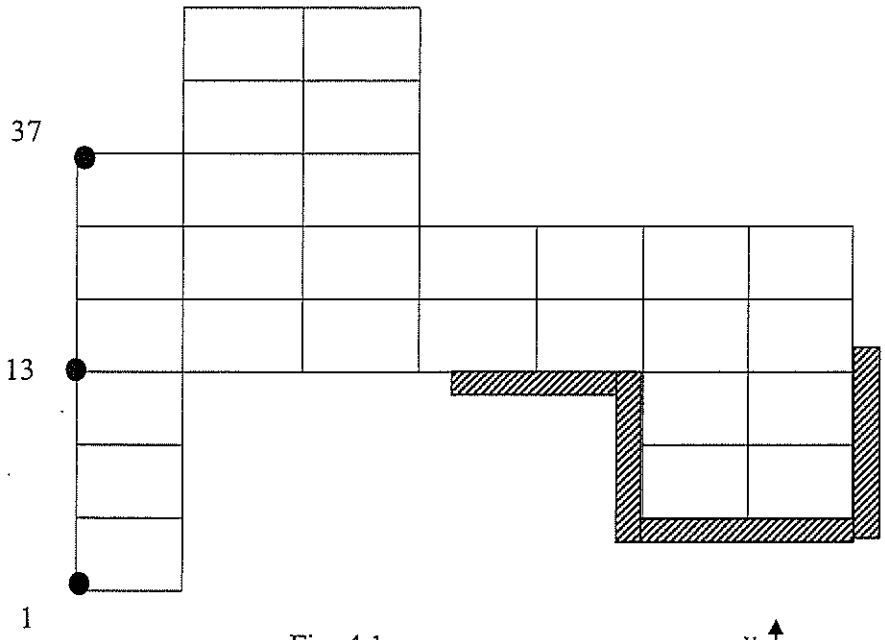
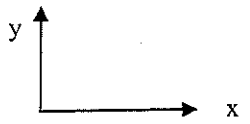


Fig. 4.1



For the highlighted area,

insulated

Node 1, 3, 8, 13, 21, 29, 37, 38, 41 and 44 are at temperature 60°C
Node 14, 15, 16, 32, 33, 34, 35, 40, 43, 45 and 46 are at temperature 57°C
Node 28 and 36 are at temperature 55°C
Node 2, 4, and 9 are at temperature 58°C
Node 5, 6, 7, 10, 12, 17, 18 and 20 are insulated

Continued...

a) Fig. 4.1 shows the temperature distribution at the boundaries of a metal plate. The metal plate domain consists of 28 rectangles with each of them of the size 2 cm x 2 cm. The domain above is discretized into equally distanced nodes on each edge on the x-y plane. Node 1 is considered as the origin of the *x-y* coordinate system and it is labeled in Fig 4.1. Nodes 13 and 37 are also labelled for you. Copy the figure into your answer sheet, and label all the missing nodes. The labeling of the nodes start from left to right and then goes from bottom to top.

[3 marks]

b) By using central difference of the second derivative, find the finite difference form of the equation (4.1). [9 marks]

c) Rewrite equation (4.1) in matrix form given by:

$$[A]\{u\} = \{c\} \tag{4.2}$$

where $\{u\}$ is a vector for nodal variable at a two-dimensional location and $\{c\}$ is the vector after incorporating the temperature at the boundaries. $[A]$ is a square matrix. DO NOT ATTEMPT TO SOLVE IT but only reproduce the Laplace equation (4.1) in matrix form. [13 marks]

Continued...

Question 5

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \quad , 0 \le x \le 1.0 \qquad (5.1)$$

Equation (5.1) is a one-dimensional heat equation and it can be used to model temperature distribution measured along a homogeneous rod. The constant α is the ratio of heat conduction to the heat storage. The independent variables are x and t.

A problem is to be modeled using heat equation above. α is taken as 1 while the spatial domain is to be discretized into six equally spaced nodes. Use time interval of $\Delta t = 0.02$ for calculation. The initial and boundary conditions are given by

$$u(x,0) = -\frac{1}{2}\sin 3\pi x + \frac{3}{2}\sin \pi x, \quad 0 \le x \le 1.0$$
$$u(0,t) = u(1.0,t) = 0$$

a) Equation (5.1) is to be solved using explicit finite difference method. Convert equation (5.1) into finite difference form using central difference scheme.
[3 marks]

b) Present the finite difference form (from part (a)) in matrix form, as given by the equation (5.2) below..

$$\{u\}^{t+1} = [A]\{u\}^t + \{\text{Boundary Conditions }\} \qquad (5.2)$$

$\{u\}^{t+1}$ and $\{u\}^t$ are vectors for nodal variables at time t+1 and t respectively. $[A]$ is a square matrix. [4 marks]

c) Solve for vector for nodal variables $\{u\}^1$ and $\{u\}^2$ at time t+1 and t +2 .
[10 marks]

d) State the stability criteria for Implicit finite difference method? [2 marks]

e) The analytical solution is given by $u(x,t) = \frac{3}{2}\sin \pi x e^{-\pi^2 t} - \frac{1}{2}\sin 3\pi x e^{-9\pi^2 t}$.
Compare the analytical solution with that from part (c) at $t = 0.04$. Comment on the accuracy of the numerical calculation. How would you increase the accuracy of your numerical calculation ?
[6 marks]

Continued...

Appendix

Explicit Euler method

$$y_{i+1} = y_i + h\, f(x_i, y_i)$$

Implicit Euler method

$$y_{i+1} = y_i + h\, f(x_{i+1}, y_{i+1})$$

Modified Euler method (predictor-corrector)

Step 1: $K_1 = f(x_i, y_i),\ y_{i+1}^P = y_i + hK_1$

Step 2: $K_2 = f(x_{i+1}, y_{i+1}^P),\ y_{i+1}^C = y_i + h\frac{1}{2}(K_1 + K_2)$

Midpoint method

Step 1: $K_1 = f(x_i, y_i),\ y_m = y_i + \frac{h}{2}K_1$

Step 2: $K_2 = f(x_m, y_m),\ y_{i+1} = y_i + hK_2$

Fourth-order Runge-Kutta

Step 1: $K_1 = f(x_i, y_i),\ y_{m1} = y_i + \frac{h}{2}K_1$

Step 2: $K_2 = f(x_m, y_{m1}),\ y_{m2} = y_i + \frac{h}{2}K_2$

Step 3: $K_3 = f(x_m, y_{m2}),\ y_{m3} = y_i + hK_3$

Step 4: $K_4 = f(x_{i+1}, y_{m3}),\ y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

Second-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$$

Third-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{12}(23f_i - 16f_{i-1} + 5f_{i-2})$$

Fourth-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{24}(55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3})$$

Second-order Adams-Moulton

$$y_{i+1} = y_i + \frac{h}{2}(f_i + f_{i+1})$$

Third-order Adams-Moulton

$$y_{i+1} = y_i + \frac{h}{12}(5f_{i+1} + 8f_i - f_{i-1})$$

Continued...

Fourth-order Adams-Moulton

$$y_{i+1} = y_i + \tfrac{h}{24}\left(9f_{i+1} + 19f_i - 5f_{i-1} + f_{i-2}\right)$$

First derivative, forward difference

$$y' = \tfrac{1}{h}\left(y_{i+1} - y_i\right)$$

First derivative, backward difference

$$y' = \tfrac{1}{h}\left(y_i - y_{i-1}\right)$$

First derivative, central difference

$$y' = \tfrac{1}{2h}\left(y_{i+1} - y_{i-1}\right)$$

Second derivative, forward difference

$$y'' = \tfrac{1}{h^2}\left(y_i - 2y_{i+1} + y_{i+2}\right)$$

Second derivative, backward difference

$$y'' = \tfrac{1}{h^2}\left(y_{i-2} - 2y_{i-1} + y_i\right)$$

Second derivative, central difference

$$y'' = \tfrac{1}{h^2}\left(y_{i-1} - 2y_i + y_{i+1}\right)$$

Jacobi method

$$x_i^{k+1} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^k - \sum_{j=1+1}^{n} a_{ij}x_j^k\right)$$

Gauss-Seidel method

$$x_i^{k+1} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=1+1}^{n} a_{ij}x_j^k\right)$$

Successive over-relaxation method

$$x_i^{k+1} = (1-\omega)x_i^k + \frac{\omega}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=1+1}^{n} a_{ij}x_j^k\right)$$

Jacobian matrix

$$[J] = \begin{bmatrix} \frac{\partial f_i}{\partial x_1} & \frac{\partial f_i}{\partial x_2} & \cdots & \frac{\partial f_i}{\partial x_n} \\ \frac{\partial g_i}{\partial x_1} & \frac{\partial g_i}{\partial x_2} & \cdots & \frac{\partial g_i}{\partial x_n} \\ \frac{\partial h_i}{\partial x_1} & \frac{\partial h_i}{\partial x_2} & \cdots & \frac{\partial h_i}{\partial x_n} \\ \vdots & \vdots & \cdots & \vdots \end{bmatrix}$$

End of page